Darwinian model risk and reverse stress testing

Stéphane Crépey,
Université de Paris / LPSM

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Partly based on **Claudio Albanese, S.C.** and **Stefano Iabichino**, ***A Darwinian Theory of Model Risk***, Risk Magazine, July 2021 pages 72-77, & ***Capital and collateral simulation for reverse stress testing***, Chapter 17 in T. Bellini, M. Eichhorn, and D. Mayenberger (eds.), Reverse Stress Testing in Banking, De Gruyter, 2021.

Introduction

- Financial derivatives:

  trade-specific pricing,
  high-quality pricing models

  $\xrightarrow{\text{2008 crisis}}$ netting-set CVA analytics,
  low-quality pricing models

  $\xrightarrow{\text{FRTB}}$ emphasis on ***model risk***

- Difference between the intrinsic value of an instrument and its model price (Derman, 2000)

Darwinian model risk

Reverse stress testing

Accounting model risk

Darwinian principles

- The better models are the ones that are econometrically more realistic and require less recalibrations over time
- In order for a bad model to remain in use for a long time and affirm itself as a market standard, it ought to satisfy two conditions
- The first one is related to competitive pricing:

  First Darwinian Principle *A lower-quality model surviving the test of time must over-value (under-value) a derivative product bought (sold) by the bank.*

- Initial mis-pricing means systematic time-decay on the portfolio valuation side as the model is re-calibrated through time
- The first Darwinian principle raises a profitability puzzle: how a systematic valuation alpha-leakage of the position can be sustained over long time horizons?
- **Second Darwinian Principle** *A surviving lower-quality model must generate systematic gains on the hedge side of the position which surpass the systematic losses on the valuation side, at least in the short to medium term.*

- But the hedging side of the position is mark-to-market, hence a martingale under the fair valuation model (worsening liquidity considerations apart), so systematic gains on the hedging side can only be the compensator of a negative jump (loss) in the future:

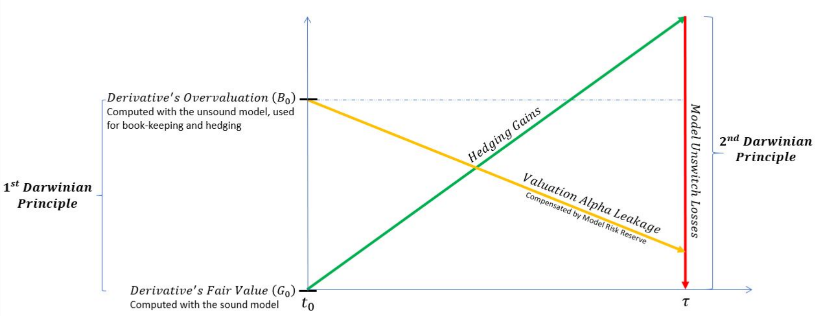

Figure: Model risk stylized pattern (in red PnL losses, in green PnL profits, in orange model risk reserve)

Toy Example: Vulnerable Put

- Suppose fair valuation corresponds to the following jump-to-ruin model (in risk-neutral form with $r = 0$):

  $$dS_t = \sigma S_t dW_t - S_{t-}(dN_t - \lambda dt),$$

  where W is a Brownian motion and N is a Poisson process with first jump time τ

- Pretending a Black-Scholes world in his trading, a trader shorts a vanilla put as a static hedge against a long position in a vulnerable put with payoff $(K - S_T)^+ \mathbb{1}_{\{\tau > T\}}$

- Before τ, the trader assigns the same value to both puts (overvaluing the vulnerable put as if its vulnerability was immaterial), so that the (hedged) PnL p of the trader vanishes.
- If $\tau \leq T$, then, at τ, the vanilla put has value K, while the vulnerable put expires worthless, hence a pnl downward jump $p_\tau - p_{\tau-} = -K\mathbb{1}_{\{\tau \leq T\}}$.
- $dp_t = \mathbb{1}_{\{t \leq \tau \wedge T\}}\big(- \lambda K dt - K\mathbb{1}_{\{\tau \leq T\}}(dN_t - \lambda dt)\big).$

VaR, Expected Shortfall and Stressed VaR myopia

- Model risk derives from the cumulative effect of daily recalibrations and feeds into the ***first moment*** of returns (negative alpha leakage).
- VaR (quantile of the possible losses), Expected Shortfall or Stressed VaR models do not detect alpha leakages because they focus on ***higher moments*** of return distributions and on ***short-time horizons*** (such as one day).
- Model risk can only be seen by ***simulating the hedging behavior of a bad model within a good model***

The case of structured products

- bank buys convexity long-term (such as 30 years) from investors with the purpose of ***selling out-of-the-money hedges trading at a premium in the market***, as these are used for risk management purposes
- model risk related to ***callability*** features, i.e. the possibility for the bank to redeem the structure early, inserted for the purpose of yield enhancement
- Risk magazine reported that Q4 of 2019 (even before covid), a ***$70bn notional*** of range accrual had to be unwound by the industry

Callable range accrual case study

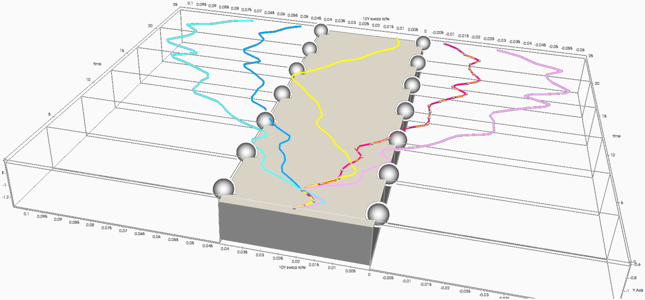

Figure: Range accrual corridor (the grey box), static hedges (on the sides of the corridor), and sample paths.

- Champion, a one-factor Gaussian model with deeply negative interest rates
- Challenger, a two-factor model with mostly positive interest rates and a stochastic short term long term skew

Seen from the Champion viewpoint:

- valuable zero floors (digital swaptions struck at 0) that you have to write
- large positive gamma of the range accrual

  $\rightarrow$ easy to hedge

- a dream trade?

That's what model risk is about...

- Champion implying higher hedge ratios and higher prices than Challenger

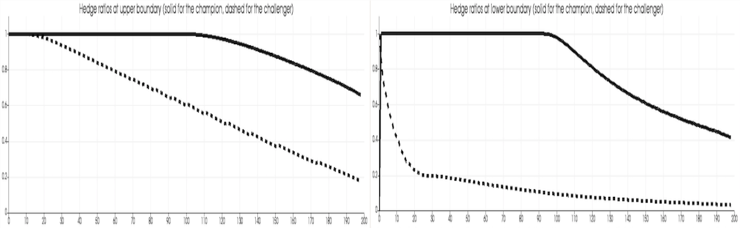

Figure: Comparison of the hedge ratios for Champion and Challenger.

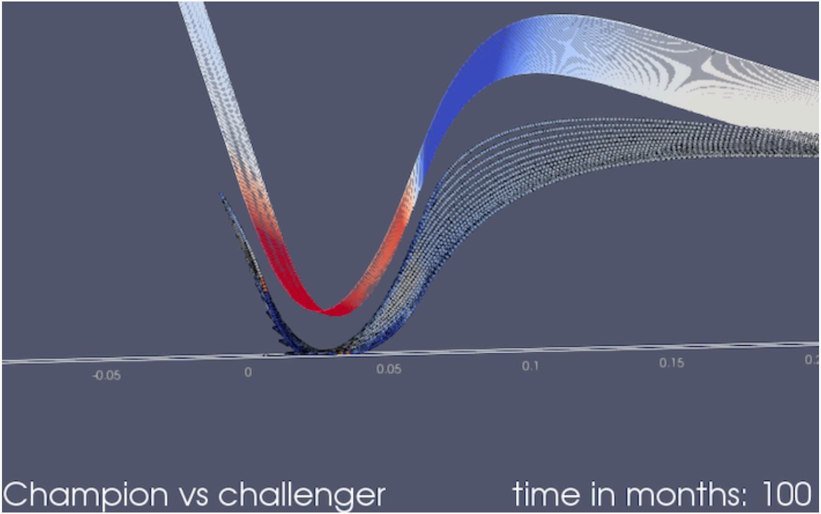

Figure: Comparison between the valuations of Challenger and Champion, color-coded for the gamma (red for positive gamma, white for intermediate, blue for negative).

"Hairs" pointing in the direction of the lowest, negative cross-gamma eigenvalues, telling you how to hedge in the Challenger two-factor model

- directions dangerous for hedging as delta hedging then goes against you

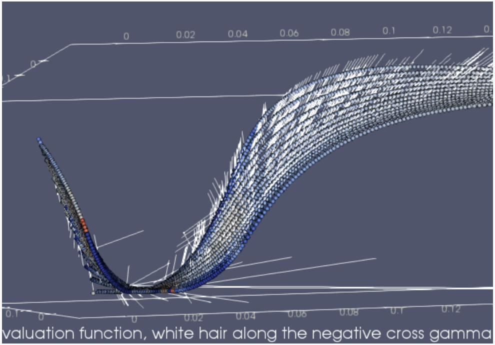

Figure: Valuations of Challenger. The white hairs point in the direction of the negative crossgamma eigenvector.

- Seen from the Challenger viewpoint, the product should be called when the interest rate in the model hits 0, which is delayed by the Champion
  - after typically 10 years by the Challenger, vs. 20 by the Champion
- When rates go to 0, the swaptions that you wrote appreciate and you end up with a large loss

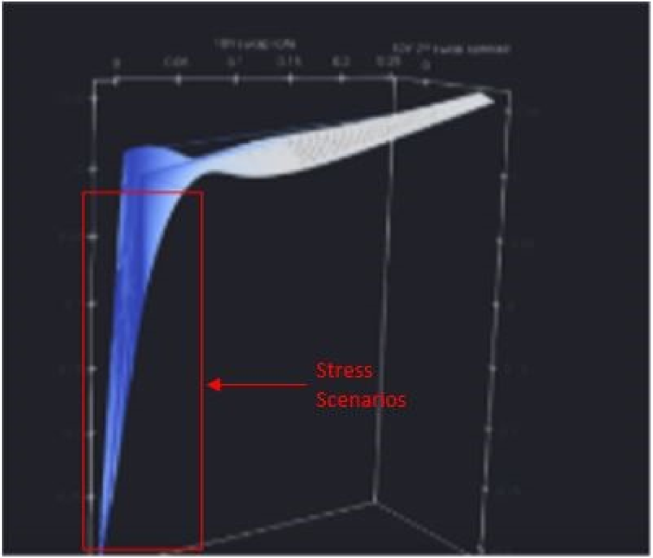

Figure: Valuation of excessive hedges shortened for a callable range accrual priced with the Champion (blue for neg. values, white for intermediate).

Darwinian model risk

Reverse stress testing

Accounting model risk

- Most model risk methodologies are backward looking
  - "what went bad", post mortem time series analysis
  - "why fix it if it's not broken"
- Instead, one should anticipate
- Complementing the state-space analysis of the first part of the talk by a ***quantification of the probabilities*** of the extreme risk events

→ ***Forward-looking reverse stress testing methodology*** based on long-term, intensive simulations

- ***Need an actionable metric*** in this context
- A rigorous FVA-KVA treatment stands at the foundation of our approach to the subject
- Ultimately one has to maximize shareholder value. Even funding strategies with debt are targeted to this ultimate objective.

$\rightarrow$ The perfect optimization target is the present value of future dividend streams, i.e. the ***KVA***

KVA

- Hedged loss-and-profit L
  - whether this residual loss L is market risk, counterparty credit risk, funding risk,...
- Economic capital projections

$$\mathrm{EC}(t) = \mathbb{E}\left[L_{t+1} - L_t \mid L_{t+1} - L_t \geq \mathrm{VaR(t)}\right], \qquad (1)$$

  where $\mathrm{VaR}(t) = \inf\{y; \mathbb{R}(L_{t+1} - L_t \leq y) \geq 97.5\%\}$.
- "Regulatory" probability measure $\mathbb{R}$ (with $\mathbb{E} = \mathbb{E}^{\mathbb{R}}$) such that (Dybvig, 1992; Artzner et al., 2020) *(i)* $\mathbb{R}_{|\mathcal{B}} = \mathbb{Q}$ and *(ii)* $\mathbb{R} = \mathbb{P}$ conditionally on $\mathcal{B}$, where $\mathcal{B}$ is the financial sub-σ-algebra of the full model σ-algebra $\mathcal{A}$ also accomodating unhedgeable risk
- (time-0) KVA $= \mathbb{E}^{\mathbb{R}} \int_0^\infty he^{-hs}\mathrm{EC}(s)ds$, for some hurdle rate $h \geq 0$ (e.g. 10%)

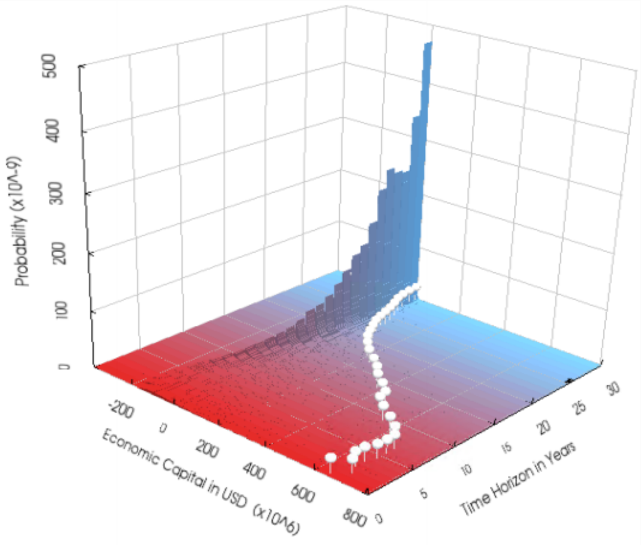

Figure: Histograms of the bank's losses $(L_{t_{i+1}} - L_{t_i})$ over time and the economic capital projections.

RST Engine

- carry out a portfolio-wide, large scale (possibly nested) simulation
    - 20K outer paths $\times$ 50 time points $\times$ 1K inner paths $\sim$ ***1bn simulation nodes***
- compute the time 0 KVA and identify stress scenarios ω as the ones contributing the most to cost of capital, as assessed by ***KVA scenario differentials*** δ_ωKVA.
- drill-down analysis of stress scenarios
    - identify the points in time where the most substantial capital depletions occur
    - gather the vector collecting the corresponding states of each economic factor

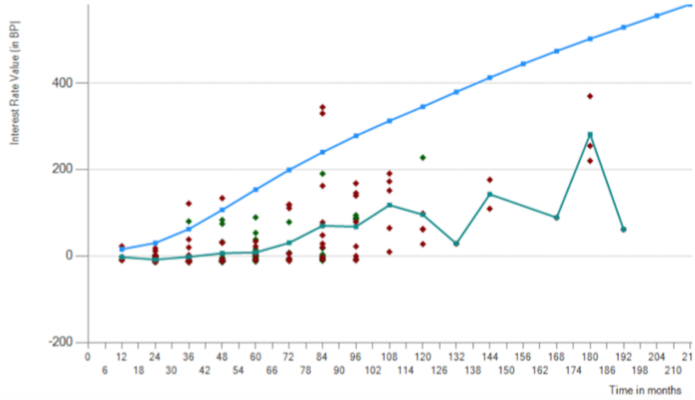

Figure: USD OIS overnight rate (in BPS) for stress scenarios on a large fixed-income derivative portfolio, ***using Challenger as interest rate model***. The blue and green lines represent the forward curve and the forward curve conditional to stress scenarios (average of the dots), respectively.

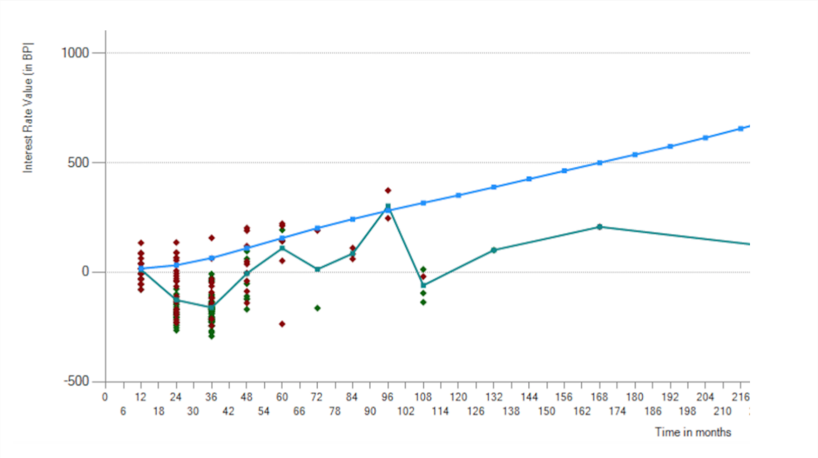

Figure: Same as before, but ***using Champion instead of Challenger as interest rate model***.

- Champion clusters the bulk of the portfolio risk in the short term.
- In contrast, Challenger disperses the risk more homogeneously over the lifetime of the OTC book.
- ***Following Champion***, extreme scenarios are deep, unrealistic, negative rates.
- In contrast, ***following Challenger***, extreme states are low but positive or nearly positive rates.

- ***systemic*** vs. ***idiosyncratic stress*** scenarios
- ***hedges*** for idiosyncratic stress scenarios
  - long-run hedges that reduce cost of capital, quite different in spirit from short-term delta hedges

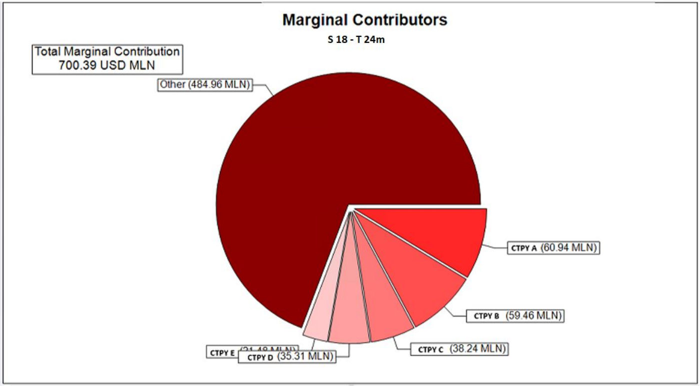

Figure: Top 5 marginal contributors to an extreme stress scenario.

- ***trading limits*** based on the counterparty incremental KVA, much more risk sensitive than the counterparty potential future exposure (PFE)
    - fully collateralized derivative portfolios result in a zero PFE although there is collateral funding risk
- ***liquidity risk management*** by shifting from the KVA to the FVA reference metric

We view reverse stress testing as a third stage of evolution for mathematical methods in finance:

- ***pricing*** is about the calculation of (risk-neutral) ***averages***,
  - cost of replication
- ***risk measures*** calculate ***tail-conditional expectations*** of the losses that arise due to imperfect hedging,
- reverse stress testing focuses on individual scenarios and their impact on risk measures

Darwinian model risk

Reverse stress testing

Accounting model risk

- ***Accounting methods are models*** in the sense of the Fed SR Letter 11-7 because “they produce numbers, are based on assumptions, and have an impact on strategies”
- ***(Darwinian) model risk*** is a concept that does not apply only to pricing models but extends to the accounting metrics.

XVA metrics at the test of the Covid-19

- The 2008 crisis triggered a shift from trade-specific pricing to ***netting-set*** **CVA** analytics
- The 2013-2016 XVA debate revolved around the definition of suitable **FVA** (cost of funding) and **KVA** (cost of capital) metrics
- https://www.**bis**.org/topic/coronavirus.htm :

  The coronavirus (Covid-19) pandemic is a major disruptive event for the global economy. It is revealing financial vulnerabilities and testing the post-financial crisis economic system.

- https://www.risk.net/derivatives/7526696/**fva-losses-back-in-spotlight**-after-coronavirus-stress:

  > *Banks face a new round of losses after two key inputs for calculating funding costs for uncollateralised derivatives – interest rates and funding spreads – saw wild moves last month, contributing to* ***a combined loss of almost \$2 billion at Bank of America, Goldman Sachs and JP Morgan***

- the Covid-19 financial crisis clearly revealed the inappropriateness of using netting-set aggregation typical to CVA analytics in an FVA and KVA context that requires broader ***funding-set*** or even ***balance-sheet wide*** metrics

- Let s_b and $x_c = x_c(\omega)$ respectively denote the bank credit spread and the (possibly negative) debt of client c to the bank in the scenario ω.
- The economically correct cost of funding formula, which should be used both for decision taking and as a capital deduction by the bank, is the asymmetric funding set FVA $= s_b \mathbb{E}[(\sum_c x_c)^+]$.
- Instead, banks are calculating their FVA numbers aggregating over netting sets (i.e. clients) c.

- Such choice does not reflect the economics of collateral management.
- It is only justified by the desire to arrive at the numbers by simply retrofitting CVA calculators, which are based on distributed computing and are performed netting set by netting set, often with netting set specific approximations.

Specifically, instead of FVA $= s_b \mathbb{E}[(\sum x_c)^+]$:

- For all their decision taking purposes, such as hedging and executives compensation (i.e. bonuses), banks use $s_b \mathbb{E} \sum_c x_c = s_b \mathbb{E} \sum_c (x_c^+) - s_b \mathbb{E} \sum_c (x_c^-) =$ FCA $-$ FBA.
- As a capital deduction, they use the FCA number.
  - Indeed, regulators insist that only asymmetric FVA numbers be used for the purpose of calculating a capital deduction.
  - They do not specify the aggregation level which could be at the netting set or funding set level and they are indifferent since the smaller is the level of aggregation, the larger and more conservative is the size of capital deduction.

In normal times:

- equity capital buffers are large enough to absorb the conservative capital deduction.
- banks' balance sheets are dominated by assets, i.e. $0 < \sum_c x_c = (\sum_c x_c)^+$ holds in most scenarios, so $s_b \mathbb{E} \sum_c x_c \approx s_b \mathbb{E}[(\sum_c x_c)^+]$, i.e. $\mathrm{FCA} - \mathrm{FBA} \approx \mathrm{FVA}$.

However, during the covid-19 financial crisis:

- Mark-downs swinged bank balance sheets towards liabilities, invalidating the above approximation.
- We saw an **8-fold credit spreads widening**,
- Increasing default rates put pressure on bank capital.

As a result:

- The number FCA-FBA used for decision taking by banks went further and further from the correct one, implying erroneous hedges and executive compensation;
- The FCA number exploded and the corresponding capital reduction became needlessly punitive for banks, at the precise bad time where capital was becoming a stringent issue for banks
- The discrepancy between the (both wrong) FCA-FBA and FCA numbers increased, enhancing the corresponding misalignment of interest between the executives and the shareholders of the bank.

...a perfect storm weather, *through which only a mathematically and numerically rigorous treatment of accounting numbers, capital models and funding strategies can be of guidance...*

Artzner, P., K.-T. Eisele, and T. Schmidt (2020). No arbitrage in insurance and the QP-rule. Working paper available as arXiv:2005.11022.

Dybvig, P. (1992). Hedging non-traded wealth: when is there separation of hedging and investment. In S. Hodges (Ed.), *Options: recent advances in theory and practice*, Volume 2, pp. 13–24. Manchester University Press.