Initiative de Recherche Risques Emergents et Atypiques en Assurance (RE2A)

Academic partner(s) : Le Mans Université

Financial partner(s) : MMA

Scientific director(s) : Anis Matoussi (Le Mans Université), Alexandre Brouste (Le Mans Université)

Website : https://www.idr-re2a.eu/

RESEARCH PROGRAM DESCRIPTION

The aim of the chair RE2A is to promote research on current probabilistic and statistical issues in actuarial sciences, dealing with insurance topics related to emerging or atypical risks:

- Improvements and generalizations of fast and efficient statistical methods (linear generalized models, change-point hypothesis tests, statistics for stochastic processes as point process or Lévy processes) in actuarial science of emerging risks as construction insurance risk (ten-year guarantees) and car insurance.
- Research developments on Systemic Risk measure applied to emerging risks as construction insurance risk (ten-year guarantees).
- Other research subjects focus on the sensitivity of emerging risk indicators and on longevity risk.
- Insurance with devices (and/or IoT) for construction insurance or car insurance.

RESEARCH TEAM

Researchers

- Alexandre BROUSTE (Principal Investigators from IRA & LMU);
- Anis MATOUSSI (Principal Investigators from IRA & LMU);
- Samir BEN-HARIZ (IRA & LMU)
- Youssef ESSTAFA (IRA & LMU);
- Sarah KAAKAI (IRA & LMU);
- Wissal SABBAGH (IRA & LMU);
- Marius Soltane (UT Compiègne).
- Alain Bensoussan (University of Texas at Dallas);
- Christophe Dutang (ENSIMAG);
- Christian Farinetto (IRA & LMU);
- Caroline Hillairet (ENSAE);
- Mohamed Mrad (Sorbonne University Paris Nord);
- Tom Rohmer (INRAE).

PhD Students

- Zakaria BENSAID (IRA & LMU);
- Guillaume BROUX-QUEMERAIS (IRA & LMU);
- Lilit HOVSEPYAN (IRA & LMU).

Interns

- Cécile MALIQUE (Université d'Angers).
- Chris BASANYA (Esprit School of Business-Tunis & LMU, Double Master's degree program);
- Oumar DJIBRINE (Esprit School of Business-Tunis & LMU, Double Master's degree program).

Private partners members

- Heloise Bertrand (Actuarial Science Manager, Covéa-Affinity) ;
- Anne Dugast (MMA) ;
- Erwan Gales (Actuarial Science Manager, MMA) ;
- Pierre Gohlen (MMA) ;
- Sabrina Savarre (Actuarial Science Manager, MMA).

Other people involved in the research initiative:

- Marie-Charlotte Ait Lmqaddam (IRA Administration, IRA & LMU);
- Francis Chavanon (Engineer, Ecole Centrale de Lyon);
- Pauline Lucet (IRA Administration, IRA & LMU).

PUBLICATIONS OF THE YEAR DIRECTLY RELATED TO THE RESEARCH PROGRAM

Published

1. **S. Ben-Hariz**, **A. Brouste**, **Y. Esstafa** and **M. Soltane** Fast calibration of weak FARIMA models, ESAIM PS, 27, 156-173, 2023.

   The asymptotically fast and efficient one-step estimation procedure for the quadratic loss function is build for FARIMA time series.

2. **A. Brouste**, **C. Dutang**, **L. Hovsepyan** and **T. Rohmer** One-step closed-form estimator for generalized linear model with categorical explanatory variables, Statis- tics and Computing, 33(138), 2023.

   The asymptotically fast and efficient one-step estimation procedure is proposed for generalized linear models (GLMs) with sole categorical explanatory variables. It is based on our previous works that build an fast initial guess closed-form estimator for GLMs. It has found an application in the pricing of guaranty in car insurance with our Covea-Affinity partner.

3. **A. Brouste** and **C. Farinetto** Fast and asymptotically efficient estimation in the Hawkes processes, Japanese Journal of Statistics and Data Science, 6, 361-379, 2023.

   The asymptotically fast and efficient one-step estimation procedure is build for self- excited Hawkes counting processes. Application with the building insurance data from Covea group.

4. **S. Kaakai, A. Matoussi and A. Tamtalini**. Estimation of Systemic Shortfall Risk Measure using Stochastic Algorithms, to appear in SIAM Journal on Financial Mathematics (SIFIN), hal 038711246-v3 (2023).

Systemic risk measures were introduced to capture the global risk and the corresponding contagion effects that is generated by an interconnected system of financial institutions. To this purpose, two approaches were suggested. In the first one, systemic risk measures can be interpreted as the minimal amount of cash needed to secure a system after aggregating individual risks. In the second approach, systemic risk measures can be interpreted as the minimal amount of cash that secures a system by allocating capital to each single institution before aggregating individual risks. Although the theory behind these risk measures has been well investigated by several authors, the numerical part has been neglected so far. In this paper, we use stochastic algorithms schemes in estimating MSRM and prove that the resulting estimators are consistent and asymptotically normal. We also test numerically the performance of these algorithms on several examples applied to insurance claims.

Published (ANRs and related works)

1. L. Belzile, **C. Dutang**, P. Northrop and T. Opitz, A modeler's guide to extreme value software, Extremes, 2023.

2. **S. Ben-Hariz**, **A. Brouste**, C. Cai and **M. Soltane** Fast and asymptotically efficient estimation in an autoregressive process with fractional type noise, Statistical Planning and Inference, 232, 2024.

   The asymptotically fast and efficient one-step estimation procedure is build for autoregressive process driven by noises with long memory.

3. **G. Broux-Quemerais, S. Kaakai, A. Matoussi, W. Sabbagh** Deep learning scheme for forward utilities using ergodic BSDEs, hal-04516019v2, to appear in Probability, Uncertainty and Quantitative Risk (PUQR).

   In this paper, we present a probabilistic numerical method for a class of forward utilities in a stochastic factor model. For this purpose, we use the representation of dynamic consistent utilities with mean of ergodic Backward Stochastic Differential Equations (eBSDEs) introduced by Liang and Zariphopoulou in [27]. We establish a connection between the solution of the ergodic BSDE and the solution of an associated BSDE with random terminal time τ, defined as the hitting time of the positive recurrent stochastic factor V. The viewpoint based on BSDEs with random horizon yields a new characterization of the ergodic cost λ which is a part of the solution of the eBSDEs. In particular, for a certain class of eBSDEs with quadratic generator, the Cole-Hopf transform leads to a semi-explicit representation of the solution as well as a new expression of the ergodic cost λ. The latter can be estimated with Monte Carlo methods. We also propose two new deep learning numerical schemes for eBSDEs, where the ergodic cost λ is optimized according to a loss function at the random horizon τ or taking into account the whole trajectory. Finally, we present numerical results for different examples of eBSDEs and forward utilities along with the associated investment strategies.

4. C. Dombry and **Y. Esstafa**. Behavior of linear L2-boosting algorithms in the vanishing learning rate asymptotic, to appear in ESAIM PS.

5. **C. Dutang**, G. Spedicato and Q. Guibert Adjusting Manual Rates to Own Ex- perience: Comparing the Credibility Approach to Machine Learning, to appear in Variance.

6. F.-B. Cartiaux, V. Le Corvec, J. Semiao and **A. Brouste**. Bridge Weigh-in-Motion: Feedback on various types of bridges, in Life-Cycle of Structure and Infrastructure Systems, CRC Press, 2023.

   IoT devices assessment are summarized for further applications in building insur- ance.

7. **Y. Esstafa**, C. Kokonendji and S. Somé. Asymptotic properties of the normalised discrete associated-kernel estimator for probability mass function, Journal of Non- parametric Statistics, 35(2), 355-372, 2023.

8. R. Dumitrescu, R. Elie, **W. Sabbagh** and C. Zhou. A new Mertens decomposition of $Y^{g,\xi}$-

submartingale systems. Application to BSDEs with weak constraints at stopping times, Stochastic Processes and their Applications, 164(C), 183-205, 2023.

We first introduce the concept of -submartingale systems, where the nonlinear op- erator corresponds to the first component of the solution of a reflected BSDE with generator and lower obstacle . We first show that, in the case of a left-limited right- continuous obstacle, any -submartingale system can be aggregated by a process which is right-lower semicontinuous. We then prove a Mertens decomposition, by using an original approach which does not make use of the standard penalization tech- nique. These results are in particular useful for the treatment of control/stopping game problems and, to the best of our knowledge, they are completely new in the literature. As an application, we introduce a new class of Backward Stochastic Differential Equations (in short BSDEs) with weak constraints at stopping times, which are related to the partial hedging of American options. We study the wellposedness of such equations and, using the Mertens decomposition, we show that the family of minimal time-values , with a supersolution of the BSDE with weak constraints, admits a representation in terms of a reflected backward stochastic differential equation.

9. **A. Matoussi**, M. Mnif, Ch. Ziri. Linear Quadratic Control Problems for Mean Field Stochastic Differential Equation with Jumps: Application in Exhaustible Re- sources Production, to appear in Stochastics, hal-03815082-v2 (2023).

   In this paper, we are interested by a stochastic model of production of an exhaustible resource, such as oil. It is known that such reserves are depleted resources, but there is a possibility of exploration and discovery of new reserves which ensure the accumulating or the upkeep of this reserves' level. We modelled the new discoveries by a jump process with intensity given by the exploration effort. We employed a weak formulation of the standard martingale optimality principle to solve a linear quadratic stochastic control problem for mean field stochastic differential equation with jumps in both cases: finite and infinite horizon.

Under review

1. Y. Boubacar Ma¨ınassara and **Y. Esstafa**. Estimation of the asymptotic variance matrix of the least-squares estimator of weak FARIMA models, in revision.

2. Y. Boubacar Ma¨ınassara, **Y. Esstafa** and B. Saussereau. Generalized least-squares estimation of fractional autoregressive models, in revision.

3. Y. Boubacar Ma¨ınassara, **Y. Esstafa** and B. Saussereau. Diagnostic checking in FARIMA models with uncorrelated but non-independent error terms, in revision.

4. **A. Brouste**, N. Dugué and W. Meskini. Speeding up the training of neural networks with the one-step procedure, submitted.

   The one-step estimation procedure for multi-layer perceptron is empirically studied in this paper.

5. **A. Brouste**, L. Denis and T. Ngo. Efficient estimation for stochastic differential equations driven by a stable Lévy process, in revision.

   The LAMN property of the parameter estimation of a SDE driven by stable Lévy process is proved with one-step asymptotically efficient estimation.

6. D. Giorgi, **S. Kaakai** and V. Lemaire. IBMPopSim : An R package for simulating Individual-Based-Models. https://daphnegiorgi.github.io/IBMPopSim/articles/IBMPopSim.html (2020).

   The R Package IBMPopSim (https://daphnegiorgi.github.io/IBMPopSim/) aims to simulate the random evolution of heterogeneous populations using stochastic Individual- Based Models (IBMs). The package

enables users to simulate population evolution, in which individuals are characterized by their age and some characteristics, and the population is modified by different types of events, including births/arrivals, death/exit events, or changes of characteristics. The frequency at which an event can occur to an individual can depend on their age and characteristics, but also on the characteristics of other individuals (interactions). Such models have a wide range of applications. For instance, IBMs can be used for simulating the evolution of a heterogeneous insurance portfolio with selection or for validating mortality forecasts.
IBMPopSim overcomes the limitations of time-consuming IBMs simulations by implementing new efficient algorithms based on thinning methods, which are compiled using the Rcpp package while providing a user-friendly interface

7. **S. Kaakai, A. Matoussi and A. Tamtalini**. Multivariate Optimized Certainty Equivalent Risk Measures and Their Numerical Computation. hal-03817818v2 (2022).

   We present a framework for constructing multivariate risk measures that is inspired from univariate Optimized Certainty Equivalent (OCE) risk measures. We show that this new class of risk measures verifies the desirable properties such as convexity, monotonocity and cash invariance. We also address numerical aspects of their computations using stochastic algorithms instead of using Monte Carlo or Fourier methods that do not provide any error of the estimation. We also test numerically the performance of this class of Multivariate Optimized Certainty Equivalent Risk Measures on several examples applied to insurance claims.

8. **S. Kaakai, A. Matoussi and A. Tamtalini**. Utility Maximization Problem with Uncertainty and a Jump Setting. hal-03813812v3 (2022).

   We study a robust utility maximization problem in the unbounded case with a general penalty term and information including jumps. We focus on time consistent penal- ties and we prove that there exists an optimal probability measure solution of the robust problem. Then, we characterize the dynamic value process of our stochastic control problem as the unique solution of generalized Quadratic-Exponential BSDE.

9. S. Mahdi, A. Verma, **C. Dutang**, P. Kiener, and J. Nash. A Review of R Neural Network Packages (with NNbenchmark): Accuracy and Ease of Use, submitted.

Working papers and other publications

(a) L. Abbas Turki, S. Crépey, S. Bouazza and **W. Sabbagh**, Pathwise XVAs: The Direct Scheme. Preprint (2022),

Motivated by the equations of cross valuation adjustments (XVAs) in the realistic case where capital is deemed fungible as a source of funding for variation margin, we introduce a simulation scheme for a class of anticipated BSDEs, where the coefficient entails a conditional expected shortfall of the martingale part of the solution. The scheme is explicit in time and uses neural network least-squares and quantile regressions for the embedded conditional expectations and expected shortfall computations. An a posteriori Monte Carlo validation procedure allows assessing the regression error of the scheme at each time step. The superiority of this scheme with respect to Picard iterations is illustrated in a high-dimensional and hybrid market/default risks XVA use-case. https://perso.lpsm.paris/ crepey/papers/MABSDE.pdf

(b) **A. Bensoussan**, **A. Brouste**, M.-A. Morlais, F.-B. Cartiaux, V. Le Corvec, J. Semiao and A. Ehrlacher. Stochastic maintenance for a large fleet of structures, hal-03918289, submitted.

Optimal randomized strategy and approximation algorithm are proposed in this paper for a stochastic control maintenance problem. It enters in the thematic of insurance with devices.

(c) **A. Brouste** and **Y. Esstafa**. One-step corrected projected stochastic gradient descent for

statistical estimation.

(d) **A. Brouste** and I. Votsi, Confidence interval for for risk indicators in semi- Markov models in the large sample scheme. *We extend our previous results to the MTTF and DTIHT indicators in the large sample scheme for semi-Markov models.*

(e) **A. Brouste**, C. Dutang and T. Rohmer, Multivariate Generalized Linear Mod- els in the case of categorical explanatory variables. *We extend our previous results for multivariate GLMs with categorical explanatory variables in order to price the house insurance.*

(f) **G. Broux-Quemerais, A. Matoussi, C. Zhou** Optimal investment and consumption under forward utilities with relative performance concerns.

In this paper, we study a n-player and mean-field portfolio optimization problem under relative performance concerns with non-zero volatility, for wealth and consumption. The consistency assumption defining forward relative performance processes allows to give a sufficient characterization of such processes with mean of a HJB-SPDE, which highlights the link between wealth and consumption utility. We investigate the existence and uniqueness of a concave so- lution to this equation by associating it to two SDEs. In particular for CRRA utilities, the HJB-SPDE allows to characterize the utility from consumption with the one from wealth. The case of separable forward performance process in power form is tractable and we establish closed form of the Nash equilibrium for both the n-player and mean field problems. We also provide some numerical examples.

(g) L. Denis, **A. Matoussi**, C. Zhou. Second order BSDEs with jumps by mea- surable selection arguments. Preprint (2022).

We prove existence and uniqueness for solution of second order BSDEs with jumps (2BSDEJs). More precisely, our problem of interest consists in the op- timization, over a set of possibly non-dominated probability measures, of sol tions of backward stochastic differential equations with jumps (BSDEJs). After proving a dynamic programming principle for this control problem in an abstract setting, we obtain a wellposedness result for second order BSDEJs (as introduced in Kazi-Tani, Possamai and Zhou (2014)) which does not require any regularity assumption on the terminal condition and the generator. This work is motivated by application for Principal-Agent problem under model un- certainty and jump-setting market.

(h) L. Denis, **A. Matoussi**, F. Noubiagain. Generalized Reflected second order BSDEs. Preprint (2022).

Our aim in this paper is to provide existence and uniqueness of Second order RBSDEs (2RBSDEs) in a general filtration without any regularity condition on generator, terminal condition and lower obstacle. This work is motivated by application for pricing and hedging American option under model uncertainty.

(i) R. Elie, **A. Matoussi**. Super-replication and Backward Stochastic Differential equation with Drawn down constraint.

We study a new class of BSDEs with drawn down constraint with application for super-replication problem for some insurance portfolio.

(j) **C. Hillairet**, T. Mastrolia and **W. Sabbagh**. Cyber risk management under optimal hacking with impulse control.

Cyber risk is a major concern for public entities and private companies, and constitutes a systemic threat to the resilience of the financial and economic world. 1% of the world' s GDP, or $1,000 billion, goes up every year because of cyber-crime. Cyberattacks are now the biggest threat to the financial system, says Jerome Powell, Chairman of the Federal Reserve global. In this paper, we develop a first study in which a cluster owner aims to protect a computer network by regularly updating or by purchasing security software against cyber- attacks. On the one hand, not protecting the computer network induces non- negligible financial losses for the owner of the cluster. On the other hand, cyber attacks can infect the network and lead to significant cyber incidents for the cluster owner and the customers of the service provided. Epidemiological models allow us to

determine an optimal level of protection against effective hacking taken as a worst-case scenario. Then, we solve optimization problems by using deep learning methods.

(k) A. Manai, **A. Matoussi**. Stability of Utility Maximization problem : A BSDE point of view.
In the present work, we consider the utility maximization problem with ad- ditional liability and under portfolio constraints to study the stability of the associated value function when the markets are nonequivalent in continuous and discontinuous frameworks. More precisely, we rely on dynamic programming principle and BSDEs techniques when the generator is convex to study the stability of the value function.

(l) **A. Matoussi**, F. Noubiagain, **W. Sabbagh**, C. Zhou. Numerical approxima- tion for Reflected second order BSDEs. Preprint (2022).
Our main contribution is to prove weak approximation of a class of second order reflected BSDE, by using an approximation of a sequence of discrete time martingale convergent to the canonical process. An auxiliary result is to extend the stability results for the case of reflected BSDEs driven by a martingale in a general filtration.

(m) **A. Matoussi**, R. Salhi. Exponential Quadratic BSDEs with infinite activity jumps. arXiv:1904.08666v2 (2019).
In this paper, we study a Backward Stochastic Differential Equation with Jumps (BSDEJs in short) where the jumps have infinite activity. Following a forward approach based on Exponential Quadratic semimartingale, we prove the existence of solution of Quadratic BSDEJs with unbounded terminal condition and quadratic growth in z. As application, we study the optimization problems of portfolio choice and indifference valuation, under ambiguity and fairly general averse preferences.
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(n) **A. Matoussi**, R. Salhi. Generalized BSDE with jumps and stochastic quadratic growth, hal-03091716 (2020).
In this paper, we study a doubly Reflected Backward Stochastic Differential Equation with jumps (DRBSDEs in short) when the driver have general exponential stochastic quadratic growth.

MAJOR COMMUNICATIONS RELATED TO THE RESEARCH PROGRAM

Major academic conferences, invited speaker, etc.

- Matoussi, invited speaker to Workshop - Advances in Quantitative finance and MF Control, National University of Singapour, 13-15 June 2023, "Stochas- tic algorithms for systemic risk measures".

- Mini-symposium Fast and asymptotically efficient inference for large and high frequency data, ICIAM, Tokyo, August 2023 with Y. Kutoyants, Youssef Es- stafa, Samir Ben-Hariz, Laurent Denis, Hiroki Masuda, Elise Bayraktar, Ahmed Kebaier, Gr´egoire Szymansky, Mikko Pakkanen, Tetsuya Takabatake, Mathieu Rosenbaum and A. Brouste.

- Matoussi, invited for two weeks at Fudan University and Shanghai Jiao Tong University, 12-23 October 2023, Shanghai, China.

Events organized by the program

- Workshop **Stochastic Control & Risk**, Hammamet, April 24-27, 2023 organized by CMAP-Ecole Polytechnique, ETH-Zürich and IRA-Le Mans University.

- Conference of the IdR RE2A **Extreme Risk & Reinsurance**, October 10, 2023, Le Mans University, with invited speakers: Julie BARBE (Responsable de l'´équipe actuariat dans la Direction des cessions en réassurance, Covéa), Sandrine BOULET (Responsable des risques Acceptations en Réassurance et des Risques Majeurs dans la Direction générale Risques, Covéa), Antoine HER- ANVAL (Coordinateur des enseignements d'actuariat et de finance `a l'ENSAE Paris), Gilles STUPFLER (Professeur à l'Université d'Angers)

- Conference to mark **the 10th anniversary of the Le Mans Risk and Insurance Institute (IRA)**, November 30 to December 1, 2023, Le Mans University.

OTHER HIGHLIGHTS

Awards, scientific recognition, organization of calls for projects, involvement in master's courses, PhD program visiting researchers, etc...

Accreditation of the Le Mans actuarial school by the Institute of Actuaries to award the title of actuary to Master of Actuarial Science graduates from June 2026.